ECE 313: Electromagnetic Waves

Lecture 8: fields in Dielectric and Good conductors

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wave propagation in dielectric:

$$j\gamma = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\sqrt{\sqrt{1+(\frac{\varepsilon''}{\varepsilon'})^2}-1} + j\omega\sqrt{\frac{\mu\varepsilon'}{2}}\sqrt{1+\sqrt{1+(\frac{\varepsilon''}{\varepsilon'})^2}} = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon_0\varepsilon_r}{2}}\sqrt{\sqrt{1+(\frac{\sigma}{\omega\varepsilon_0\varepsilon_r})^2}-1}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon_0\varepsilon_r}{2}}\sqrt{\sqrt{1+(\frac{\sigma}{\omega\varepsilon_0\varepsilon_r})^2}+1}$$

Example:

A sinusoidal electric intensity of amplitude 250 (V/m) and frequency 1 (GHz) exists in a lossy dielectric medium that has a relative permittivity of 2.5 and a loss tangent of 0.001. Find the average power dissipated in the medium per cubic meter.

Solution First we must find the effective conductivity of the lossy medium:

$$\tan \delta_c = 0.001 = \frac{\sigma}{\omega \epsilon_0 \epsilon_r},$$

$$\sigma = 0.001(2\pi 10^9)\left(\frac{10^{-9}}{36\pi}\right)(2.5)$$

$$= 1.39 \times 10^{-4} \text{ (S/m)}.$$

$\varepsilon_0 = 8.85*10^{-12} = 10^{-9}/(36\pi)$

The average power dissipated per unit volume is

$$p = \tfrac{1}{2}JE = \tfrac{1}{2}\sigma E^2$$

$$= \tfrac{1}{2} \times (1.39 \times 10^{-4}) \times 250^2 = 4.34 \quad (\text{W/m}^3).$$

Example 11.4

consider plane wave propagation in water, at microwave frequency of 2.5 GHz.
At frequencies in this range and higher. Real and imaginary parts of the permittivity are present., and both vary with frequency. ϵ'' that increases with increasing frequency, ϵ' decreases with increasing frequency. At 2.5 GHz, $\epsilon'_R = 78$ and $\epsilon''_R = 7$

$$\alpha = \frac{(2\pi \times 2.5 \times 10^9)\sqrt{78}}{(3.0 \times 10^8)\sqrt{2}} \left(\sqrt{1 + \left(\frac{7}{78}\right)^2} - 1 \right)^{1/2} = 21 \text{ Np/m}$$

similar to that for α, we find $\beta = 464$ rad/m. The wavelength is $\lambda = 2\pi/\beta = 1.4$ cm, whereas in free space this would have been $\lambda_0 = c/f = 12$ cm.

$$\eta = \frac{377}{\sqrt{78}} \frac{1}{\sqrt{1 - j(7/78)}} = 43 + j1.9 = 43\angle 2.6^\circ \ \Omega$$ and E_x leads H_y in time by 2.6° at every point.

$$\eta = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon(1 - j\frac{\varepsilon''}{\varepsilon})}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \sqrt{\frac{1}{1 - j\frac{\varepsilon''}{\varepsilon}}} \qquad \textit{where} \qquad \varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon', \frac{\varepsilon''}{\varepsilon} = \frac{\varepsilon''_r}{\varepsilon_r}$$

Example:

Let the internal dimension of a coaxial capacitor be $a = 1.2$ cm, $b = 4$ cm, and $l = 40$ cm. The homogeneous material inside the capacitor has the parameters $\epsilon = 10^{-11}$ F/m, $\mu = 10^{-5}$ H/m, and $\sigma = 10^{-5}$ S/m. If the electric field intensity is $\mathbf{E} = (10^6/\rho)\cos(10^5 t)\mathbf{a}_\rho$ V/m, find:

a) $\mathbf{J}$: Use

$$\mathbf{J} = \sigma\mathbf{E} = \underline{\left(\frac{10}{\rho}\right)\cos(10^5 t)\mathbf{a}_\rho \ \text{A/m}^2}$$

b) the total conduction current, I_c, through the capacitor: Have

$$I_c = \int\int \mathbf{J}\cdot d\mathbf{S} = 2\pi\rho l J = 20\pi l\cos(10^5 t) = \underline{8\pi\cos(10^5 t)\ \text{A}}$$

c) the total displacement current, I_d, through the capacitor: First find

$$\mathbf{J}_d = \frac{\partial\mathbf{D}}{\partial t} = \frac{\partial}{\partial t}(\epsilon\mathbf{E}) = -\frac{(10^5)(10^{-11})(10^6)}{\rho}\sin(10^5 t)\mathbf{a}_\rho = -\frac{1}{\rho}\sin(10^5 t)\ \text{A/m}$$

Now

$$I_d = 2\pi\rho l J_d = -2\pi l\sin(10^5 t) = \underline{-0.8\pi\sin(10^5 t)\ \text{A}}$$

d) the ratio of the amplitude of I_d to that of I_c, the quality factor of the capacitor: This will be

$$\frac{|I_d|}{|I_c|} = \frac{0.8}{8} = \underline{0.1}$$

Example 27.3 The Radial Resistance of a Coaxial Cable

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with polyethylene plastic as shown in Figure 27.8a. Current leakage through the plastic, in the *radial* direction, is unwanted. (The cable is designed to conduct current along its length, but that is *not* the current being considered here.) The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer conductor is $b = 1.75$ cm, and the length is $L = 15.0$ cm. The resistivity of the plastic is $1.0 \times 10^{13}\ \Omega \cdot \text{m}$. Calculate the resistance of the plastic between the two conductors.

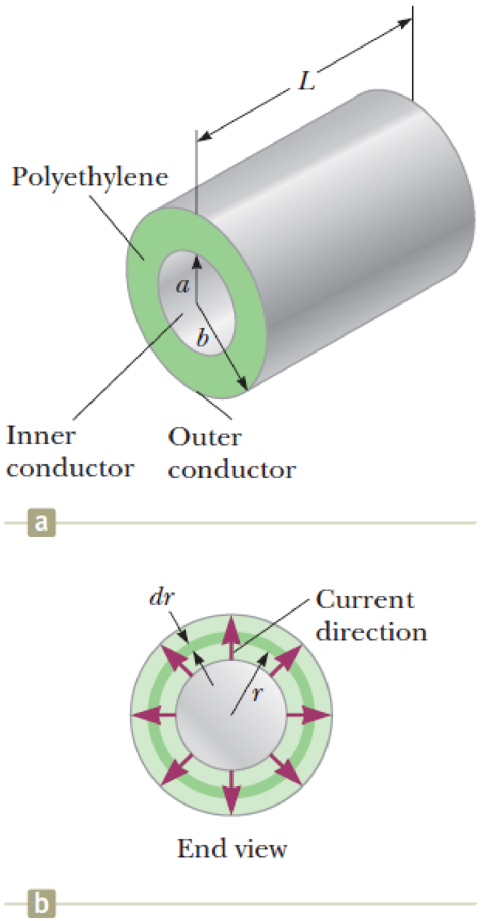


Figure 27.8 (Example 27.3) A coaxial cable. (a) Polyethylene plastic fills the gap between the two conductors. (b) End view, showing current leakage.

SOLUTION

Conceptualize Imagine two currents as suggested in the text of the problem. The desired current is along the cable, carried within the conductors. The undesired current corresponds to leakage through the plastic, and its direction is radial.

Categorize Because the resistivity and the geometry of the plastic are known, we categorize this problem as one in which we find the resistance of the plastic from these parameters. Equation 27.10, however, represents the resistance of a block of material. We have a more complicated geometry in this situation. Because the area through which the charges pass depends on the radial position, we must use integral calculus to determine the answer.

Analyze We divide the plastic into concentric cylindrical shells of infinitesimal thickness dr (Fig. 27.8b). Any charge passing from the inner to the outer conductor must move radially through this shell. Use a differential form of Equation 27.10, replacing ℓ with dr for the length variable: $dR = \rho\ dr/A$, where dR is the resistance of a shell of plastic of thickness dr and surface area A.

Analyze We divide the plastic into concentric cylindrical shells of infinitesimal thickness dr (Fig. 27.8b). Any charge passing from the inner to the outer conductor must move radially through this shell. Use a differential form of Equation 27.10, replacing ℓ with dr for the length variable: $dR = \rho\, dr/A$, where dR is the resistance of a shell of plastic of thickness dr and surface area A.

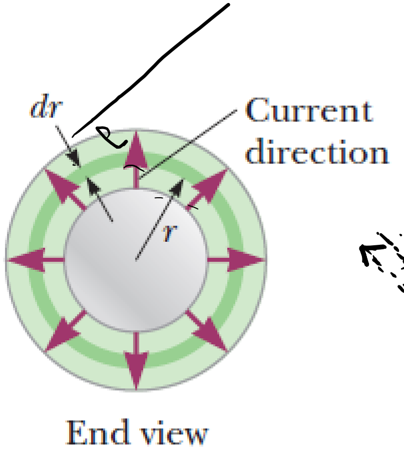
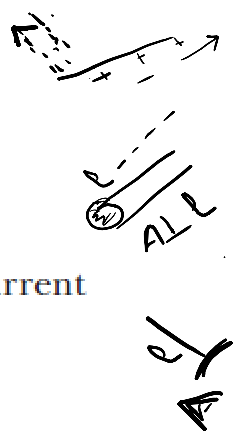


Figure End view, showing current leakage.



Write an expression for the resistance of our hollow cylindrical shell of plastic representing the area as the surface area of the shell:

$$dR = \frac{\rho\, dr}{A} = \frac{\rho}{2\pi r L}\, dr$$

Integrate this expression from $r = a$ to $r = b$:

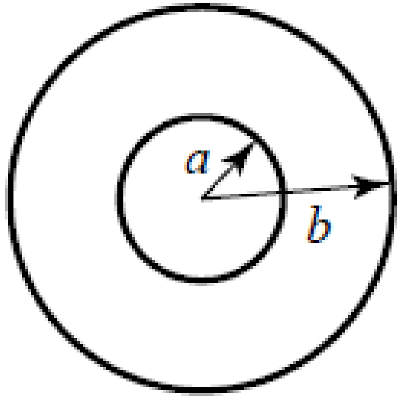
$$(1)\quad R = \int dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

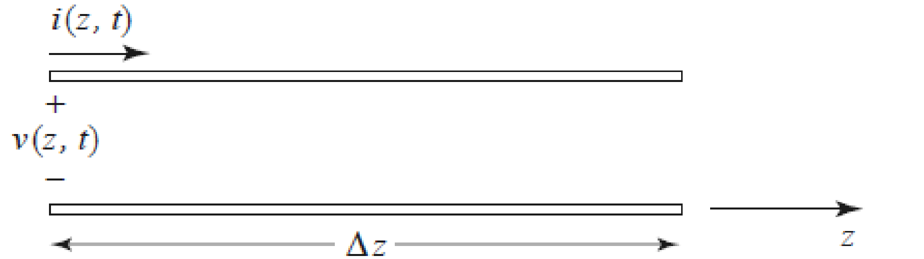
Substitute the values given:

$$R = \frac{1.0 \times 10^{13}\ \Omega \cdot \mathrm{m}}{2\pi(0.150\ \mathrm{m})}\ln\left(\frac{1.75\ \mathrm{cm}}{0.500\ \mathrm{cm}}\right) = \boxed{1.33 \times 10^{13}\ \Omega}$$

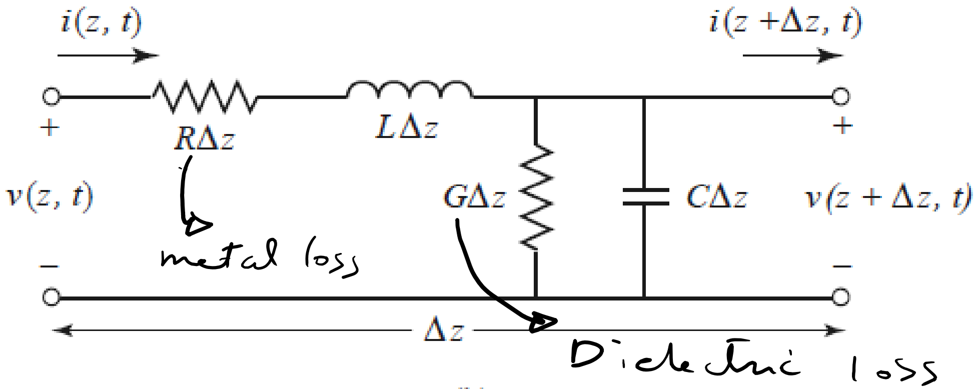
$$R = \frac{1}{2\pi L \sigma}\ln\left(\frac{b}{a}\right) \longrightarrow \frac{G}{L} = \frac{2\pi\sigma}{\ln(b/a)} \quad \Omega^{-1}/m$$

COAX





(a)



(b)

R $$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right),\ R_s=\frac{1}{\delta\sigma}$$

G $$\frac{2\pi\omega\epsilon''}{\ln b/a}=\frac{2\pi\sigma}{\ln(b/a)}$$

Example:

If Teflon is coaxial dielectric material compute its conductivity, Complex permittivity, phase velocity, ratio of conduction current to displacement current and penetration depth

APPENDIX G DIELECTRIC CONSTANTS AND LOSS TANGENTS FOR SOME MATERIALS

Material	Frequency	ϵ_r	$\tan\delta$ (25°C)
Teflon	10 GHz	2.08	0.0004

$$\tan\delta = \frac{\sigma}{\omega\epsilon_0\epsilon_r} \rightarrow \sigma = \omega\epsilon\tan\delta = 2\pi\times10^{10}\times8.85\times10^{-12}\times2.08\times4\times10^{-4} = 4.6\times10^{-4}$$

$$\epsilon_c = \epsilon' - j\epsilon'' = 8.85\times10^{-12}\times2.08 - j\frac{4.6\times10^{-4}}{2\pi\times10^{10}} = 18.4\times10^{-12} - j73.6\times10^{-16}$$

compute β for $v_{ph} = \omega/\beta$

$$j\gamma = j\omega\sqrt{\mu\epsilon_c} = j2\pi\times10^{10}\sqrt{10^{-20}}\sqrt{4\pi(18.4 - j73.6\times10^{-3})} = 0.0604 + j302.13$$

$\alpha = 0.0604$ Np/m, $\beta = 302.13$ rad/m

$$v_{ph} = \frac{2\pi\times10^{10}}{302.13} = 2.08\times10^{8} \text{ m/s}, \quad \left|\frac{J_c}{J_d}\right| = \tan\delta = 0.0004$$

$$z = 1/\alpha = 16.55 \text{ m}$$

Wave propagation in good conductors:

- Good conductors has large conduction current w.r.t displacement current ($J_c >> J_d$, or $J_c/J_d >> 1$)

Loss tangent($\tan\theta$)

$$\left|\frac{J_c}{J_d}\right| = \left|\frac{\sigma E}{\omega \varepsilon E}\right| = \boxed{\left|\frac{\sigma}{\omega \varepsilon'}\right| >> 1} \quad , \varepsilon' = \varepsilon_0 \varepsilon_r$$

$$j\gamma = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon'}} \approx j\sqrt{-j\omega\mu\sigma}$$

$$\approx \sqrt{\frac{\omega\mu\sigma}{2}}(1+j) = \alpha + j\beta \quad \rightarrow \quad \boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}} \qquad \varepsilon_c \approx -j\frac{\sigma}{\omega} = -j\varepsilon''$$

$$\sqrt{-j} = \sqrt{1\angle -90^o} = 1\angle -45^o = \frac{1-j}{\sqrt{2}}$$

Attenuation and propagation inside good conductor
Depend only on freq & conductivity

- Medium of EM travels: parameters in general formula
- Free space:

$$\sigma = 0 \quad \varepsilon = \varepsilon_0 \quad \mu = \mu_0$$

- Lossless dielectric:

$$\sigma = 0 \quad \varepsilon = \varepsilon_0 \varepsilon_r \quad \mu = \mu_0 \mu_r$$

$$or \quad \sigma << \omega\varepsilon$$

- Lossy dielectric:

$$\sigma \neq 0 \quad \varepsilon = \varepsilon_0 \varepsilon_r \quad \mu = \mu_0 \mu_r$$

- Good conductor:

$$\sigma \approx \infty \quad \varepsilon = \varepsilon_0 \quad \mu = \mu_0$$

$$or \quad \sigma >> \omega\varepsilon$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon_0\varepsilon_r}{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon_0\varepsilon_r}\right)^2}-1}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon_0\varepsilon_r}{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon_0\varepsilon_r}\right)^2}+1}$$

Lossy dielectric is the most general case

Lossy dielectric partially conducting medium

Wave propagation in good conductors:

Assume E_x travelling in +z direction

$$E_x = E_{x0} e^{-\alpha z} e^{-j\beta z} e^{j\omega t} = E_{x0} e^{-z\sqrt{\pi f \mu \sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma})$$

At boundary surface z=0

Displacement current is negligible(at good conductor)

$$E_x = E_x \cos(\omega t)$$

$$J_{cx} = \sigma E_x$$

Exponential decay to e^{-1}=0.368 of initial value(z=0) at:

$$z = \frac{1}{\sqrt{\pi f \mu \sigma}} = \delta = \frac{1}{\alpha} = \frac{1}{\beta}$$

Skin depth or depth of penetration

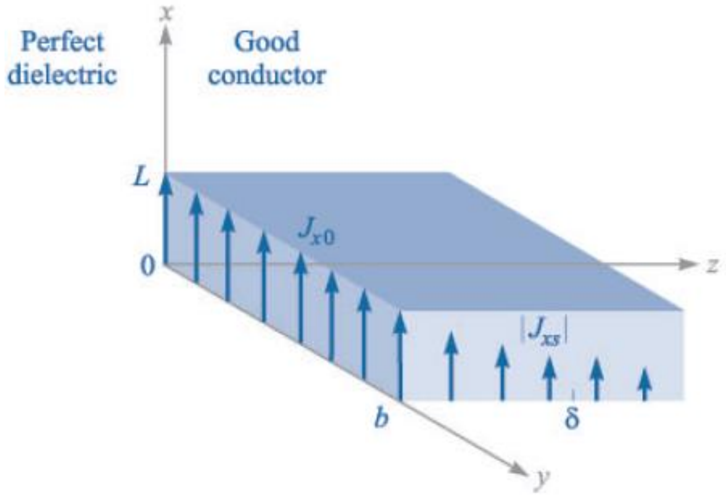


FIGURE 11.3
The current density $J_x = J_{x0}e^{-z/\delta}e^{-jz/\delta}$ decreases in magnitude as the wave propagates into the conductor. The average power loss in the region $0 < x < L$, $0 < y < b$, $z > 0$, is $\delta b L J_{x0}^2 / 4\sigma$ watts.

Skin Depth

- For cupper σ_{cu}=5.8×10^{7} S/m so skin depth δ_{cu}=8.53 mm at f=60HZ
- Electromagnetic energy is not transmitted in the interior of a conductor; it travels in the region surrounding the conductor, while the conductor guide the wave.
- As frequency increase skin depth decrease, so at microwave frequencies only 0.0001 inch coating silver for waveguide(may be glass) is excellent at these frequencies.
- So skin depth depend only on frequency and conductivity,
- We will see for submarine communication a very low frequency is required to made a communication in water.

Frequency	δ_{cu}
10 [Hz]	2.1 [cm]
100 [Hz]	6.6 [mm]
1 [kHz]	2.1 [mm]
10 [kHz]	0.66 [mm]
100 [kHz]	0.21 [mm]
1 [MHz]	66 [μm]
10 [MHz]	21 [μm]
100 [MHz]	6.6 [μm]
1 [GHz]	2.1 [μm]
10 [GHz]	0.66 [μm]
100 [GHz]	0.21 [μm]

Velocity, wavelength, intrinsic impedance inside conductor

$$\alpha = \beta = \frac{1}{\delta} = \sqrt{\pi f \mu \sigma}$$

$$\lambda = \frac{2\pi}{\beta} = 2\pi\delta$$

$$v_p = \frac{\omega}{\beta} = 2\pi f \delta$$

$for \quad copper\ at\ 60Hz \rightarrow \beta = 117.211,\ \delta_{cu} = 1/\beta = 8.53mm$

$\lambda = 2\pi \times 8.53mm = 5.36cm$

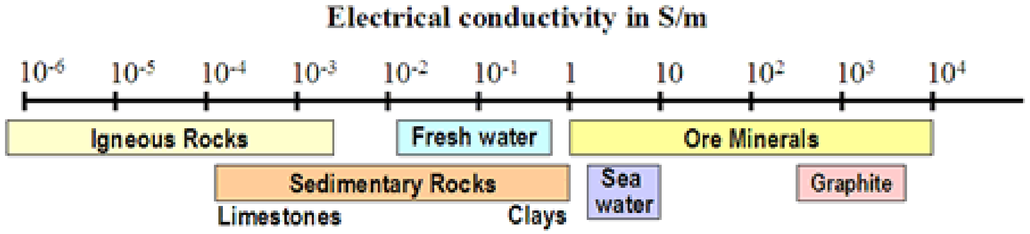
$v_p = 2\pi \times 8.53mm \times 60 = 3.215\ m/s$

$$\eta = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{j\frac{\omega\mu}{\sigma}} = \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\sqrt{\frac{\omega\mu}{\sigma}}$$

$$= \frac{\sqrt{\pi f \mu \sigma}}{\sigma}(1+j) = \frac{1}{\sigma\delta}(1+j) = \frac{\sqrt{2}}{\sigma\delta}\angle 45$$

Example 11.6

Let us again consider wave propagation in water, but this time we will consider seawater. The primary difference between seawater and fresh water is of course the salt content. Sodium chloride dissociates in water to form Na^+ and Cl^- ions, which, being charged, will move when forced by an electric field. Seawater is thus conductive, and so will attenuate electromagnetic waves by this mechanism. At frequencies in the vicinity of 10^7 Hz and below, the bound charge effects in water discussed earlier are negligible, and losses in seawater arise principally from the salt-associated conductivity. We consider an incident wave of frequency 1 MHz. We wish to find the skin depth, wavelength, and phase velocity. In seawater, $\sigma = 4$ S/m, and $\epsilon'_R = 81$.



Solution. We first evaluate the loss tangent, using the given data:

$$\frac{\sigma}{\omega\epsilon'} = \frac{4}{(2\pi \times 10^6)(81)(8.85 \times 10^{-12})} = 8.9 \times 10^2 \gg 1$$

Thus, seawater is a good conductor at 1 MHz (and at frequencies lower than this). The skin depth is

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{(\pi \times 10^6)(4\pi \times 10^{-7})(4)}} = 0.25 \text{ m} = 25 \text{ cm}$$

Now

$$\lambda = 2\pi\delta = 1.6 \text{ m}$$

and

$$v_p = \omega\delta = (2\pi \times 10^6)(0.25) = 1.6 \times 10^6 \text{ m/sec}$$

In free space, these values would have been $\lambda = 300$ m and of course $v = c$.

With a 25 cm skin depth, it is obvious that radio frequency communication in seawater is quite impractical. Notice however that δ varies as $1/\sqrt{f}$, so that things will improve at lower frequencies. For example, if we use a frequency of 10 Hz in the extremely low frequency (ELF) range, the skin depth is increased over that at 1 MHz by a factor of $\sqrt{10^6/10}$, so that

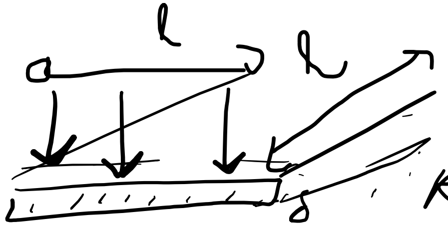
$$\delta(10Hz) \doteq 80 \text{ m}$$

The corresponding wavelength is $\lambda = 2\pi\delta \doteq 500$ m. Frequencies in the ELF range are in fact used for submarine communications, chiefly between gigantic ground-based antennas (required since the free-space wavelength associated with 10 Hz is 3×10^7 m) and submarines, from which a suspended wire antenna of length shorter than 500 m is sufficient to receive the signal. The drawback is that signal data rates at ELF are so slow that a single word can take several minutes to transmit. Typically, ELF signals are used to tell the submarine to implement emergency procedures, or to come near the surface in order to receive a more detailed message via satellite.

Surface impedance

Consider a good conductor, a plane wave normally incident on this conductor is mostly reflected, and the power that is transmitted into the conductor is dissipated as heat within a very short distance from the surface[pozar page33]

$$R_s = X_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\omega\mu}{2\sigma}}\,\Omega$$

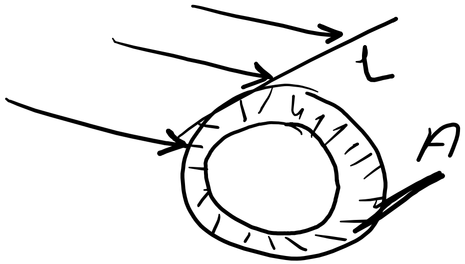


$R = \frac{\rho l}{A} = \frac{1}{\sigma\delta}\ \Omega$

$A = l\delta$

for coaxial with inner conductor radius a, outer conductor radius b

$$R_{s\,inner} = \frac{1}{2\pi a\sigma\delta},\ R_{s\,outer} = \frac{1}{2\pi b\sigma\delta},\ R_{tot} = R_{s\,inner} + R_{s\,outer}\ \Omega/m$$



$A = 2\pi a\delta$

$R = \frac{\rho l}{A} \Rightarrow \frac{1}{\sigma\, 2\pi a\delta} = \frac{R}{l}\ \Omega/m$

Example

11.22. The inner and outer dimensions of a copper coaxial transmission line are 2 and 7 mm, respectively. Both conductors have thicknesses much greater than δ. The dielectric is lossless and the operating frequency is 400 MHz. Calculate the resistance per meter length of the:

a) inner conductor: First

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi (4 \times 10^8)(4\pi \times 10^{-7})(5.8 \times 10^7)}} = 3.3 \times 10^{-6}\text{m} = 3.3\mu\text{m}$$

Now, using (70) with a unit length, we find

$$R_{in} = \frac{1}{2\pi a \sigma \delta} = \frac{1}{2\pi (2 \times 10^{-3})(5.8 \times 10^7)(3.3 \times 10^{-6})} = \underline{0.42 \text{ ohms/m}}$$

b) outer conductor: Again, (70) applies but with a different conductor radius. Thus

$$R_{out} = \frac{1}{2\pi b \sigma \delta} = \qquad R_{out} = \frac{a}{b} R_{in} = \frac{2}{7}(0.42) = \underline{0.12 \text{ ohms/m}}$$

c) transmission line: Since the two resistances found above are in series, the line resistance is their sum, or $R = R_{in} + R_{out} = \underline{0.54 \text{ ohms/m}}$.